short distances. The electrolyte concentration must be carefully controlled to allow the van der Waals forces to have their full effect. Note that in this case the droplets will be able to wet the fibers.

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Convective Diffusion through Wavy Liquid Films in Horizontal Shear Flow

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The hydrodynamics of a liquid film flowing concurrently with an adjacent gas phase are complicated by the shear stress and pressure variations which result from the interactions between the gas and liquid flows. Jeffreys (1925) first analyzed the interaction between wind and waves, and renewed interest in the phenomenon was generated by Miles (1957) and Benjamin (1959). Recent measurements of the air flow characteristics over water waves made by Chang et al. (1971) emphasize the complexity of the interaction and suggest that rigorous analysis is most difficult.

The interfacial waves encountered in gas-liquid flows can significantly affect the transport of heat or mass in the two phases and across the interface, so it is desirable to develop approximate theories of the hydrodynamics to make heat and mass transfer predictions. As indicated by Frisk and Davis (1972) in their heat transfer study with horizontal air-water flow, several investigations of two-phase flow heat or mass transfer have been made—mostly experimental. Most theoretical analyses of the hydrodynamics of film flow have been for the case of zero shear stress at the wavy surface, but Leonard and Estrin (1972) applied an extended Kapitsa approach to describe the film flow for constant interfacial shear, and they applied the result to predict heat transfer through falling liquid films. By an approximate numerical solution of the thermal energy equation, they predicted significant heat transfer enhancement through the film for large amplitude waves distorted from sinusoidal shape, using the dimensionless wave velocity predicted from the Kapitsa analysis ($C = 3$).

Experimental data on waves in horizontal gas-liquid flow obtained by Narasimhan and Davis (1971, 1972) indicate that there are two important differences between falling film flows and horizontal shear flows:

1. The quasi-laminar two-dimensional waves that exist over a narrow range of gas and liquid flow rates for horizontal flow have small wave amplitudes and short wavelengths ($\lambda \approx 1$ cm).
2. The dimensionless wave speed is considerably higher for shear flows than for film flows without interfacial shear.

It is the purpose of this paper to analyze the convective diffusion in wavy film flow with interfacial shear to elucidate the effects of wave velocity and other parameters on heat or mass transport in the liquid film. The analysis of the hydrodynamics follows that by Narasimhan and Davis (1972).

HYDRODYNAMICS

Consider the fully developed wavy flow (no growth or decay of the waves) over a horizontal surface under the influence of pressure gradient ψ and with interfacial shear τ, both considered to be functions of axial distance x. Assume that the velocity profile in the wavy film has the same form as that for smooth film flow except for the fact that ψ, τ, and the film thickness a are functions of x. Then the axial velocity distribution is given by

$$u = \frac{\psi(x)}{2\mu_l}[y^2 - 2a(x)\,y] + \frac{\tau(x)y}{\mu_l} \qquad (1)$$

Ruckenstein and Berbente (1965, 1968) pointed out that the analogous expression for falling film flow with no interfacial shear does not lead to the correct dependence of wavelength on velocity, but Narasimhan and Davis found that the method used here leads to predicted wavelengths and wave frequencies that are in excellent agreement with experimental values provided that measured wave velocities are used in the calculation of the wave parameters. Substituting Equation (1) into the equations of motion for the liquid film, assuming a periodic film thickness of the form $a = a_0(1 + \phi)$, and following Narasimhan's procedure, we obtain the following approximate equation for the wave shape function ϕ:

$$\frac{1}{We}\phi''' - \left(\frac{1}{Fr} + C_0\right)\phi' = 0 \qquad (2)$$

where C_0 approximates to $-C^2 + \frac{8}{3}C - \frac{4}{3}$, the Weber and Froude numbers are defined by We $= \rho\bar{u}_0^2a_0/\sigma$, and $Fr = \bar{u}_0^2/a_0g$, respectively, and the prime refers to the derivative with respect to $z = (x - ct)/a_0$. Equation (2) has a particular solution of the form

$$\phi = \epsilon\cos Kz = \epsilon\cos k(x - ct) \qquad (3)$$

where ϵ is the wave amplitude and $K = ka_0$ is the wave number given by

$$K = \left[We\left(C^2 - \frac{8}{3}C + \frac{4}{3} - \frac{1}{Fr}\right)\right]^{1/2} \qquad (4)$$

We consider the wave amplitude and the dimensionless wave velocity C to be experimentally measurable parameters in this formulation. It is to be expected that C is a function of the gas phase flow characteristics.

Assuming the pressure perturbation due to the waves to be 180° out of phase with the wave the dimensionless pressure gradient can be approximated by

$$\Psi = \Psi_0 - M\frac{d\phi}{dz} \qquad (5)$$

where Ψ_0 is the dimensionless pressure gradient for the undisturbed flow and M is related to the amplitude of the pressure perturbation. The pressure gradient, the interfacial shear, and the average velocity of the film can be related through a mass balance on the film to give the following approximation for the dimensionless interfacial shear:

$$\frac{\tau a_0}{\mu\,\bar{u}_0} \equiv T_i = \left(2 + \frac{2}{3}\Psi_0\right) + \left(2Z - 4 + \frac{2}{3}\Psi_0\right)\phi - \frac{2}{3}M\frac{d\phi}{dz} + 0(\phi^2) \qquad (6)$$

Since $\Psi_0 << 1$, as indicated by the measurements of Narasimhan, Ψ_0 can be neglected in Equation (6).

If it is assumed that Equations (5) and (6) apply to waves distorted from the simple cosine shape of Equation (3), we can examine the effects of wave shape as well as the other parameters on the convective diffusion. To this purpose let us generalize the wave shape by considering the following piecewise continuous function

$$\phi = \epsilon\cos(\delta z + \gamma) \qquad (7)$$

where

$$\delta z + \gamma = \begin{cases} \dfrac{\pi L a_0 z}{\lambda} & \text{for } 0 \leq z \leq \dfrac{\lambda}{2a_0L} \\ \dfrac{\pi}{L-1}\left[\dfrac{La_0}{\lambda}z + (L-2)\dfrac{\pi}{2}\right] & \text{for } \dfrac{\lambda}{2a_0L} \leq z \leq \dfrac{\lambda}{a_0}\left(1 - \dfrac{1}{2L}\right) \\ \pi\left[\dfrac{La_0}{\lambda}z + (2-L)\pi\right] & \text{for } \dfrac{\lambda}{a_0}\left(1 - \dfrac{1}{2L}\right) \leq z \leq \dfrac{\lambda}{a_0} \end{cases}$$

The wavelength λ is given by $\lambda = 2\pi/k$. For $L = 2$ the wave shape is the simple cosine, and as L increases the wave trough broadens.

CONVECTIVE DIFFUSION

The thermal energy equation and the convective diffusion equation for mass transfer have the same dimensionless form, which for heat transfer is

$$(U - C)\frac{\partial\theta}{\partial X} + VPe\frac{\partial\theta}{\partial Y} = \frac{\partial^2\theta}{\partial Y^2} + \frac{1}{Pe^2}\frac{\partial^2\theta}{\partial X^2} \qquad (8)$$

where

$$U = \frac{u}{\bar{u}_0},\quad C = \frac{c}{\bar{u}_0},\quad V = \frac{v}{\bar{u}_0},\quad \theta = \frac{T - T_s}{T_w - T_s},$$

$$Y = \frac{y}{a_0} \quad\text{and}\quad X = \frac{x - ct}{a_0Pe} \quad Pe = \bar{u}_0a_0/\alpha$$

and for heat transfer the boundary conditions are $\theta(X, 0) = 1$, $\theta(X, 1) = 0$, $\theta(X, Y) = \theta(X + m\Lambda, Y)$ where the dimensionless wave length is defined by $\Lambda = \lambda/a_0Pe$. For mass transfer the flux is zero at the wall ($Y = 0$). The axial velocity is given by

$$U = 2Y + 2(C - 2)Y\phi - \frac{M}{2}\left(Y^2 - \frac{2}{3}Y\right)\frac{d\phi}{dz} + 0(\phi^2) \qquad (9)$$

and the transverse velocity is obtained by integrating the equation of continuity to give

$$V = -(C - 2)\,Y^2\frac{d\phi}{dz} + \frac{M}{6}(Y^3 - Y^2)\frac{d^2\phi}{dz^2} + 0(\phi^2) \qquad (10)$$

For sufficiently large Peclet numbers the axial conduction term can be neglected, so we shall drop the last term in Equation (8). If the wave amplitude is not large we can anticipate that the temperature field in the liquid film will not deviate greatly from that of the undisturbed film, so let us look for a perturbation solution of the form

$$\theta = \theta^{(0)} + \epsilon\theta^{(1)} + \ldots\ldots \qquad (11)$$

It should be noted that the transverse velocity V is of order ϵ. We obtain the following balances up to first order in ϵ:

$$\epsilon^0 : \frac{\partial^2\theta^{(0)}}{\partial Y^2} = 0 \qquad (12)$$

$$\epsilon^1: \frac{\partial^2 \theta^{(1)}}{\partial Y^2} - (2Y - C)\frac{\partial \theta^{(1)}}{\partial X} = \left\{ -\left[\frac{M}{6}(Y^3 - Y^2)\,\delta^2 KW\right]\cos(\delta WX + \gamma) + [(C-2)Y^2\,\delta W]\sin(\delta WX + \gamma)\right\}\frac{\partial \theta^{(0)}}{\partial Y} \quad (13)$$

where $W = KPe$.

The boundary conditions are

$\theta^{(0)}(X,0) = 1,\ \ \theta^{(0)}(X,1) = 0,\ \ \theta^{(1)}(X,0) = \theta^{(1)}(X,1) = 0,$ and $\theta^{(1)}(X,Y) = \theta^{(1)}(X + m\Lambda, Y)$

The zeroth order solution, the solution for undisturbed smooth film flow, is simply

$$\theta^{(0)} = 1 - Y \quad (14)$$

We look for a first-order solution of the form

$$\theta^{(1)} = \sum_{m=1}^{\infty} \{g_m(Y)\sin m(\delta WX + \gamma) - f_m(y)\cos m(\delta WX + \gamma)\} \quad (15)$$

Substituting the assumed solution in Equation (13) leads to a coupled set of differential equations in the coefficients $f_m(Y)$ and $g_m(Y)$, which have been solved by a power series method to give a solution of the form

$$\theta(X,Y) = 1 - Y + \epsilon\,[-f_1(Y)\cos(\delta WX + \gamma) + g_1(y)\sin(\delta WX + \gamma)] \quad (16)$$

RESULTS

The perturbations of the temperature field are functions of the parameters W, K, C, M, and the wave shape. The parameter W provides some indication of the importance of the convective transport of heat in the axial direction compared with transverse transport. For sufficiently small wave numbers and moderate Peclet numbers W is small and the deviations from the smooth film temperature distribution should be small. At sufficiently large Peclet numbers it is to be expected that even though short wavelength ripples are present axial convection predominates, and the waves do not appreciably affect the temperature field away from the interface. It should be pointed out that the perturbation scheme is invalid for large W.

Figures 1 and 2 show typical dimensionless temperature profiles predicted for various parameters which were selected to be consistent with the experimental parameters of Narasimhan. The wave velocity and wave shape are found to have a very pronounced effect on the temperature field, and the wave number and M have much less effect than the wave amplitude and the wave velocity C.

HEAT TRANSFER ENHANCEMENT

We can define an enhancement factor to describe the effects of wave motion on the transfer process, that is, let the enhancement $\overline{E}$ be defined in terms of the average Nusselt numbers over one wavelength as follows:

$$\overline{E} = \frac{\overline{Nu} - \overline{Nu}^{(0)}}{\overline{Nu}^{(0)}} = \epsilon \overline{Nu}^{(1)}$$

where $\overline{Nu}^{(0)}$ is the Nusselt number calculated from the smooth film solution ($\overline{Nu}^{(0)} = 1$), and $\overline{Nu}^{(1)}$ is the dimensionless temperature gradient calculated from the first-order solution of the temperature field. For the interface,

$$\overline{E}_s = -\epsilon \left.\frac{\partial \theta^{(1)}}{\partial Y}\right|_{Y=1};$$

and for the wall,

$$\overline{E}_w = -\epsilon \left.\frac{\partial \theta^{(1)}}{\partial Y}\right|_{Y=0}.$$

Figure 3 shows the predicted enhancement factors $\overline{E}_w$ and $\overline{E}_s$ as functions of the various parameters involved. The wave number K and the perturbation pressure gradient amplitude factor M were found to have very little effect on enhancement. The enhancement of heat transfer between the wall and the wavy film, which is also the heat transfer rate through the film, is not significant for the type

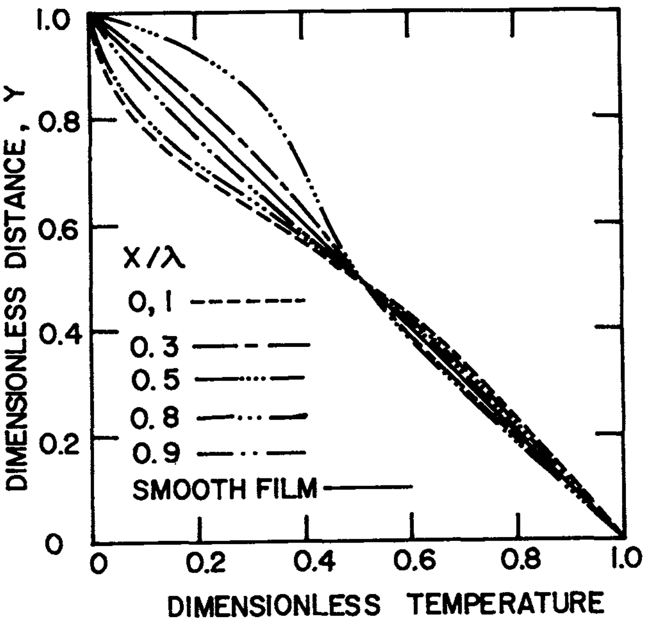

Fig. 1. Dimensionless temperature profiles for $L = 2$, $K = 0.1$, $W = 10$, $M = 30$, $C = 7$ and $\epsilon = 0.1$.

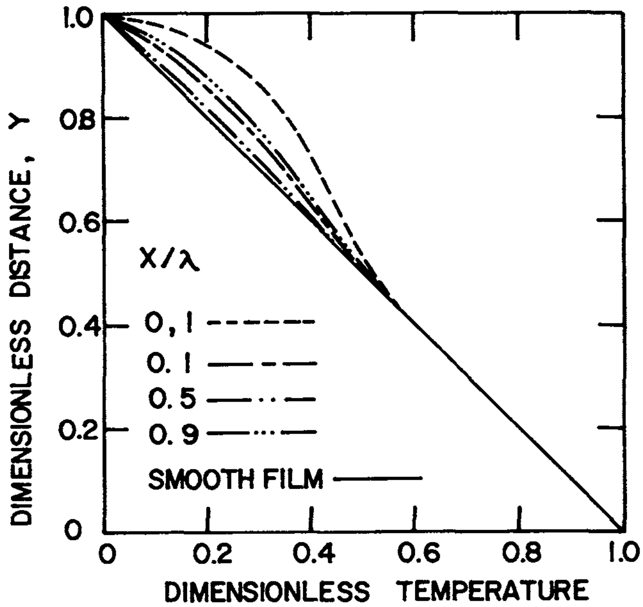

Fig. 2. Dimensionless temperature profiles for $L = 4$. $K = 0.1$, $W = 15$, $M = 30$, $C = 5$ and $\epsilon = 0.05$.

of wavy flow considered here. But the convective transport is appreciably influenced in the vicinity of the gas-liquid interface. The large values of $\overline{E_s}$ calculated provide an estimate of the rates that would be encountered when heat or mass is transferred across the interface under conditions such that small penetration occurs. However, the problem of small penetration at the free surface, which is usually the case for gas absorption, is better treated by the method of Ruckenstein and Berbente.

CONCLUSIONS

1. The heat transfer through the liquid film flowing under shear by a concurrent gas flow is not appreciably enhanced by the presence of small amplitude short wave length surface ripples. It is likely that larger enhancements which have been reported in the literature are due not to surface ripples but to turbulence associated with roll waves or the precursors of roll waves observed and discussed by Frisk and Davis.

2. The effects of surface waves penetrate to about half the depth of the liquid film, and in the upper half the predicted effects of convection produced by the wave motion are consistent with reported enhancement of mass transfer at the wavy interface.

3. The wave shape and wave velocity have a large effect on the predicted temperature distributions.

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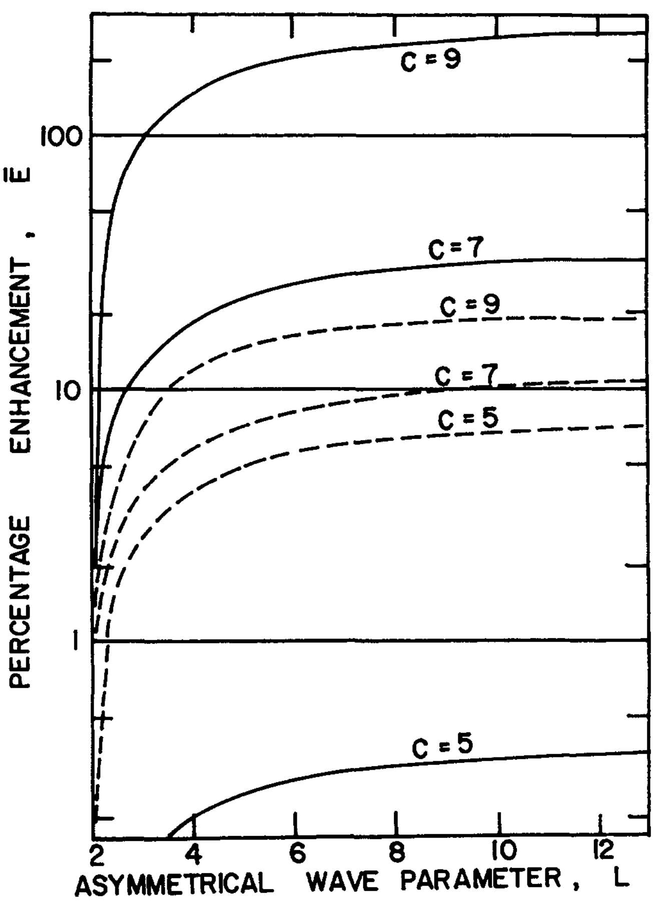

Fig. 3. Heat transfer enhancement at the wall (- - -) and at the interface (——) for various dimensionless wave velocities and for $K = 0.1, W = 10, M = 30$ and $\epsilon = 0.05$.

NOTATION

a, A = film thickness, cm and dimensionless, respectively
a_0 = average film thickness over one wavelength, cm
c, C = wave speed, cm/s and dimensionless, respectively
$f_m(Y), g_m(Y)$ = functions in Equation (15)
g = acceleration of gravity, cm/s^2
k, K = wave number, cm^{-1} and dimensionless, respectively
L = wave shape parameter
m = integer
M = parameter in Equation (5)
$\overline{Nu}$ = average Nusselt number for one wavelength, dimensionless
t = time, s
T = temperature, °K
T_i = dimensionless interfacial shear stress
u, U = axial velocity, cm/s and dimensionless, respectively
v, V = transverse velocity, cm/s and dimensionless, respectively
$W = KPe$ = dimensionless parameter
x, X = axial coordinate, cm and dimensionless, respectively
y, Y = transverse coordinate, cm and dimensionless, respectively

Greek Letters

$\alpha = k_l/\rho_l C_p$ = thermal diffusivity, cm^2/s
γ, δ = parameters in Equation (7)
ϵ = wave amplitude
θ = dimensionless temperature
λ, Λ = wavelength, cm and dimensionless, respectively
μ = viscosity, g/cm-s
ν = kinematic viscosity, cm^2/s
ρ = density, g/cm^3
σ = surface tension, dynes/cm
τ = interfacial shear stress, dynes/cm^2
ϕ = wave shape, dimensionless
ψ, Ψ = pressure gradient, dynes/cm^3 and dimensionless, respectively

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